

# Substitutions

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Équipe Automates et Applications

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# **1. Substitutions sur des mots**

1  $\mapsto$  12

2  $\mapsto$  1

1  $\mapsto$  12

2  $\mapsto$  1

1

1  $\mapsto$  12

2  $\mapsto$  1

12

1  $\mapsto$  12

2  $\mapsto$  1

121

1  $\mapsto$  12

2  $\mapsto$  1

12112

1  $\mapsto$  12

2  $\mapsto$  1

12112121



1  $\mapsto$  12

2  $\mapsto$  1

1211212112112

1  $\mapsto$  12

2  $\mapsto$  1

121121211211212112121

1  $\mapsto$  12  
2  $\mapsto$  1

1211212112112121121211211212112112112

1  $\mapsto$  12  
2  $\mapsto$  1

1211212112112121121211211212112112121121211211211

1  $\mapsto$  12

2  $\mapsto$  1

121121211211212112121121211211212112121121121211211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

1  $\mapsto$  12

2  $\mapsto$  1

121121211211212112121121211211212112121121121211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*a*

1  $\mapsto$  12

2  $\mapsto$  1

121121211211212112121121211211212112121121121211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabb*

1  $\mapsto$  12

2  $\mapsto$  1

121121211211212112121121121211211212112121121121211211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabb*



1  $\mapsto$  12

2  $\mapsto$  1

1211212112112121121211211212112112121121121211211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa*

1  $\mapsto$  12

2  $\mapsto$  1

121121211211212112121121121211212112121121121211211211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabba*

0  $\mapsto$  10

1  $\mapsto$  01

1  $\mapsto$  12

2  $\mapsto$  1

1211212112112121121211211212112112121121121211211211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa*

0  $\mapsto$  10

1  $\mapsto$  01

0

1  $\mapsto$  12

2  $\mapsto$  1

121121211211212112121121121211212112121121121211211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabba*

0  $\mapsto$  10

1  $\mapsto$  01

10

1  $\mapsto$  12  
2  $\mapsto$  1

1211212112112121121211211212112112121121211211211211

*a*  $\mapsto$  *aabb*  
*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa*

0  $\mapsto$  10  
1  $\mapsto$  01

0110

1  $\mapsto$  12  
2  $\mapsto$  1

1211212112112121121211211212112112121121211211211211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabba*

0  $\mapsto$  10

1  $\mapsto$  01

10010110

1  $\mapsto$  12

2  $\mapsto$  1

1211212112112121121211211212112112121121121121121

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa*

0  $\mapsto$  10

1  $\mapsto$  01

0110100110010110





1  $\mapsto$  12

2  $\mapsto$  1

1211212112112121121211211212112112121121121211211211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabba*

0  $\mapsto$  10

1  $\mapsto$  01

0110100110010110100101100110100110010110010110011010

1  $\mapsto$  12

2  $\mapsto$  1

12112121121121211212112112121121121211211211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa*

0  $\mapsto$  10

1  $\mapsto$  01

0110100110010110100101100110100110010110011010

*x*  $\mapsto$  *xyz*

*y*  $\mapsto$  *yz*

*z*  $\mapsto$  *xyz*

1  $\mapsto$  12

2  $\mapsto$  1

12112121121121211212112112112121121121211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa*

0  $\mapsto$  10

1  $\mapsto$  01

0110100110010110100101100110100110010110011010

*x*  $\mapsto$  *xyz*

*y*  $\mapsto$  *yz*

*x*

*z*  $\mapsto$  *xyz*

1  $\mapsto$  12

2  $\mapsto$  1

12112121121121211212112112121121121211211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa*

0  $\mapsto$  10

1  $\mapsto$  01

0110100110010110100101100110100110010110011010

*x*  $\mapsto$  *xyz*

*y*  $\mapsto$  *yz*

*z*  $\mapsto$  *xyz*

*xyz*

1  $\mapsto$  12

2  $\mapsto$  1

12112121121121211212112112121121121211211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa*

0  $\mapsto$  10

1  $\mapsto$  01

0110100110010110100101100110100110010110011010

*x*  $\mapsto$  *xyz*

*y*  $\mapsto$  *yz*

*z*  $\mapsto$  *xyz*

*xyzxyzxyzxyz*

1  $\mapsto$  12

2  $\mapsto$  1

1211212112112121121211211212112112121121211211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa*

0  $\mapsto$  10

1  $\mapsto$  01

0110100110010110100101100110100110010110011010

*x*  $\mapsto$  *xyz*

*y*  $\mapsto$  *yz*

*z*  $\mapsto$  *xyz*

*xyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyz*



1  $\mapsto$  12

2  $\mapsto$  1

1211212112112121121211211212112112121121211211211

a  $\mapsto$  aabb

b  $\mapsto$  aabb

aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa

0  $\mapsto$  10

1  $\mapsto$  01

0110100110010110100101100110100110010110011010

x  $\mapsto$  xyz

y  $\mapsto$  yz

z  $\mapsto$  xyz

xyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyz

1  $\mapsto$  12

2  $\mapsto$  13

3  $\mapsto$  14

4  $\mapsto$  1

1234



1  $\mapsto$  12

2  $\mapsto$  1

1211212112112121121211211212112112121121211211211

a  $\mapsto$  aabb

b  $\mapsto$  aabb

aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa

0  $\mapsto$  10

1  $\mapsto$  01

0110100110010110100101100110100110010110011010

x  $\mapsto$  xyz

y  $\mapsto$  yz

z  $\mapsto$  xyz

xyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyz

1  $\mapsto$  12

2  $\mapsto$  13

3  $\mapsto$  14

4  $\mapsto$  1

1213141

1  $\mapsto$  12

2  $\mapsto$  1

1211212112112121121211211212112112121121211211211

a  $\mapsto$  aabb

b  $\mapsto$  aabb

aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa

0  $\mapsto$  10

1  $\mapsto$  01

0110100110010110100101100110100110010110011010

x  $\mapsto$  xyz

y  $\mapsto$  yz

z  $\mapsto$  xyz

xyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyz

1  $\mapsto$  12

2  $\mapsto$  13

3  $\mapsto$  14

4  $\mapsto$  1

1213121412112

1  $\mapsto$  12

2  $\mapsto$  1

1211212112112121121211211212112112121121211211

*a*  $\mapsto$  *aabb*

*b*  $\mapsto$  *aabb*

*aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa*

0  $\mapsto$  10

1  $\mapsto$  01

0110100110010110100101100110100110010110011010

*x*  $\mapsto$  *xyz*

*y*  $\mapsto$  *yz*

*z*  $\mapsto$  *xyz*

*xyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyz*

1  $\mapsto$  12

2  $\mapsto$  13

3  $\mapsto$  14

4  $\mapsto$  1

1213121412131211213121213

1  $\mapsto$  12  
2  $\mapsto$  1

1211212112112121121211211212112112121121211211

a  $\mapsto$  aabb  
b  $\mapsto$  aabb

aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa

0  $\mapsto$  10  
1  $\mapsto$  01

0110100110010110100101100110100110010110011010

x  $\mapsto$  xyz  
y  $\mapsto$  yz  
z  $\mapsto$  xyz

xyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyz

1  $\mapsto$  12  
2  $\mapsto$  13  
3  $\mapsto$  14  
4  $\mapsto$  1

1213121412131211213121412131212131214121312131

1  $\mapsto$  12

2  $\mapsto$  1

1211212112112121121211211212112112121121211211

a  $\mapsto$  aabb

b  $\mapsto$  aabb

aabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaabbaa

0  $\mapsto$  10

1  $\mapsto$  01

0110100110010110100101100110100110010110011010

x  $\mapsto$  xyz

y  $\mapsto$  yz

z  $\mapsto$  xyz

xyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyzxyz

1  $\mapsto$  12

2  $\mapsto$  13

3  $\mapsto$  14

4  $\mapsto$  1

1213121412131211213121412131212131214121312112

## Substitutions, dynamique, fractals

$$\sigma : \begin{cases} 1 \mapsto 12 \\ 2 \mapsto 13 \\ 3 \mapsto 1 \end{cases}$$

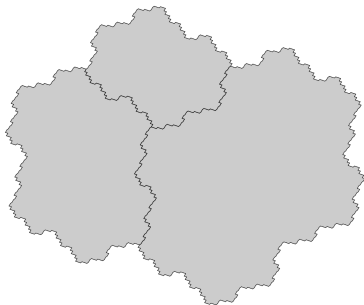
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$$\sigma : \begin{cases} 1 \mapsto 12 \\ 2 \mapsto 13 \\ 3 \mapsto 1 \end{cases}$$



# Substitutions, dynamique, fractals

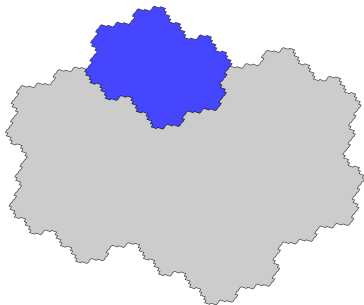
$$\sigma : \begin{cases} 1 \mapsto 12 \\ 2 \mapsto 13 \\ 3 \mapsto 1 \end{cases}$$





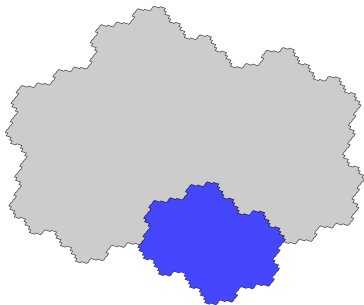
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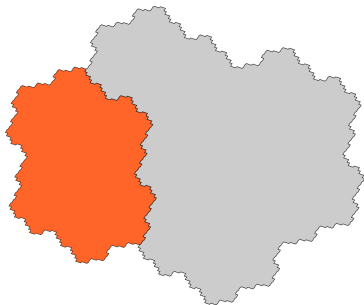
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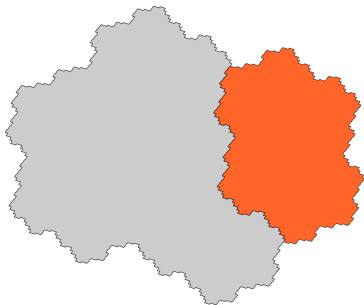
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$$\sigma : \begin{cases} 1 \mapsto 12 \\ 2 \mapsto 13 \\ 3 \mapsto 1 \end{cases}$$



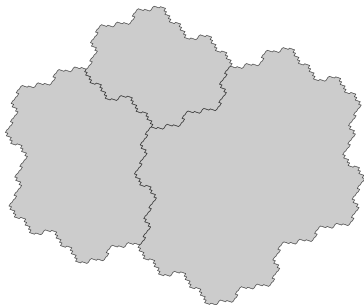
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$$\sigma : \begin{cases} 1 \mapsto 12 \\ 2 \mapsto 13 \\ 3 \mapsto 1 \end{cases}$$



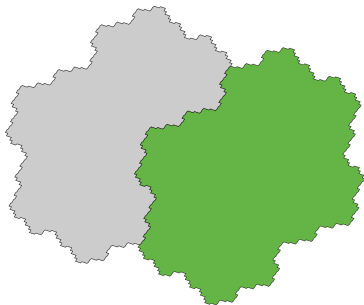
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$$\sigma : \begin{cases} 1 \mapsto 12 \\ 2 \mapsto 13 \\ 3 \mapsto 1 \end{cases}$$



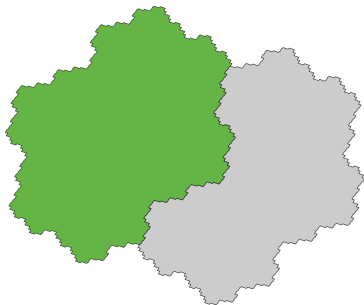
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$$\sigma : \begin{cases} 1 \mapsto 12 \\ 2 \mapsto 13 \\ 3 \mapsto 1 \end{cases}$$



# Substitutions, dynamique, fractals

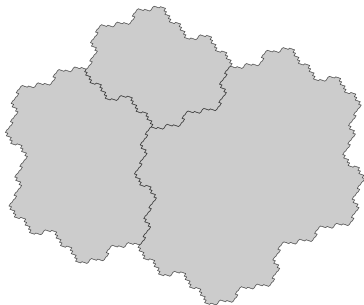
$$\sigma : \begin{cases} 1 \mapsto 12 \\ 2 \mapsto 13 \\ 3 \mapsto 1 \end{cases}$$





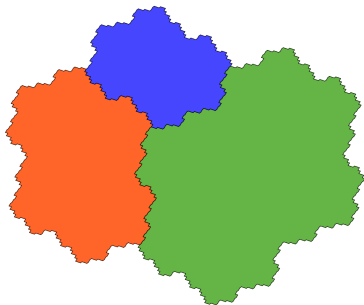
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$$\sigma : \begin{cases} 1 \mapsto 12 \\ 2 \mapsto 13 \\ 3 \mapsto 1 \end{cases}$$



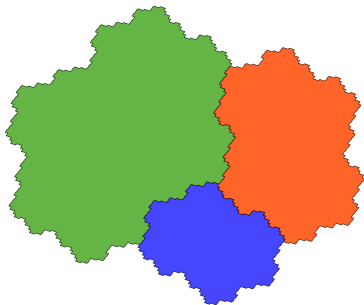
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$$\sigma : \begin{cases} 1 \mapsto 12 \\ 2 \mapsto 13 \\ 3 \mapsto 1 \end{cases}$$



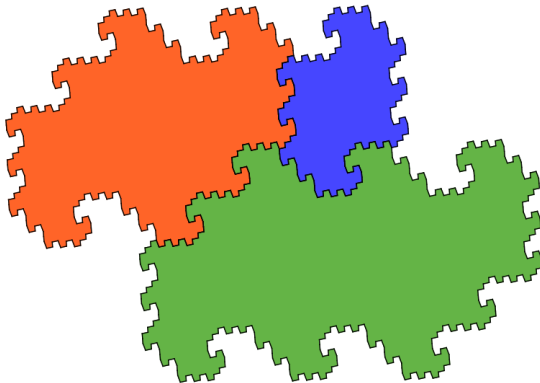
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$$\sigma : \begin{cases} 1 \mapsto 12 \\ 2 \mapsto 13 \\ 3 \mapsto 1 \end{cases}$$



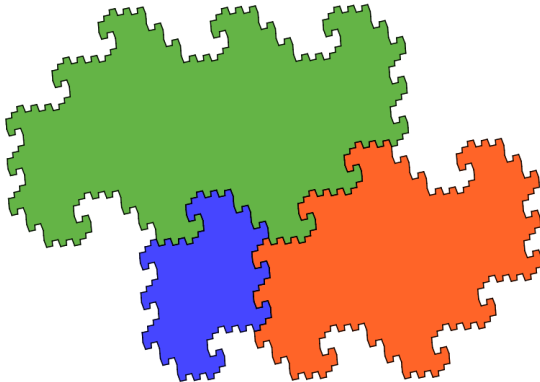
# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$



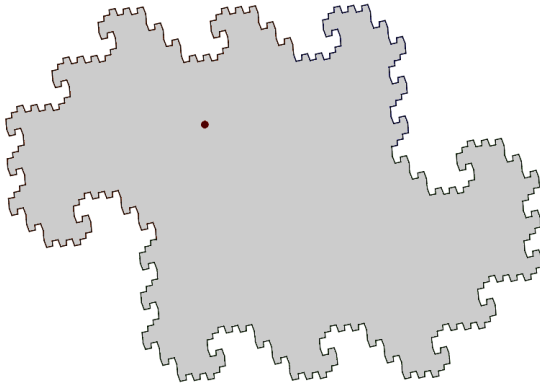
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$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$



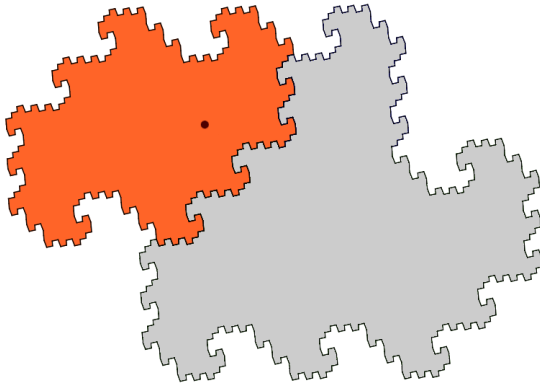
# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$



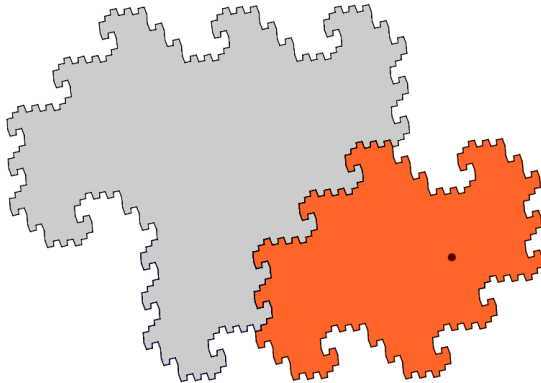
# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$



# Dynamique de l'échange de morceaux

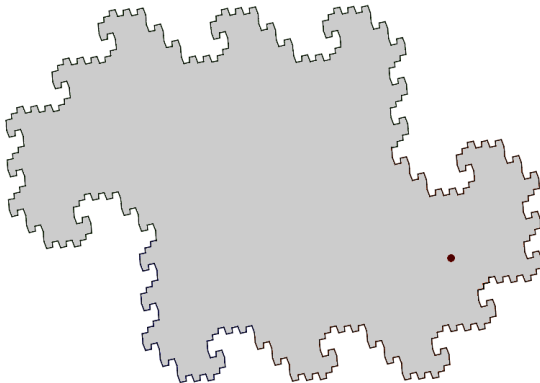
$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$





# Dynamique de l'échange de morceaux

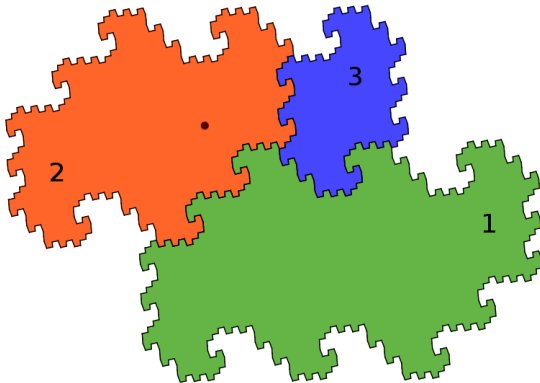
$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$



# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

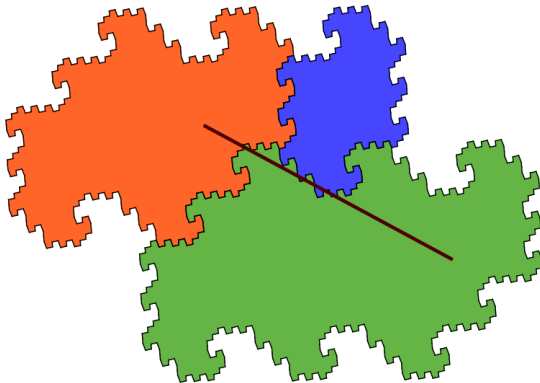
Orbite : 2



# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

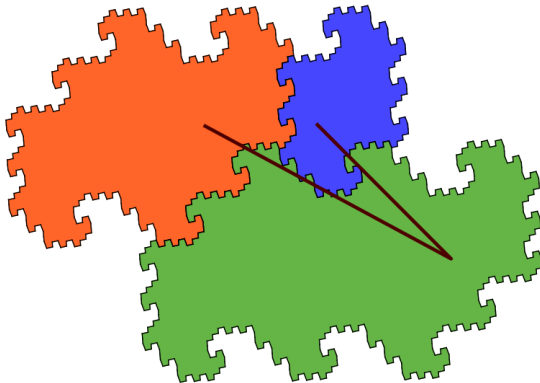
Orbite : 21



# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

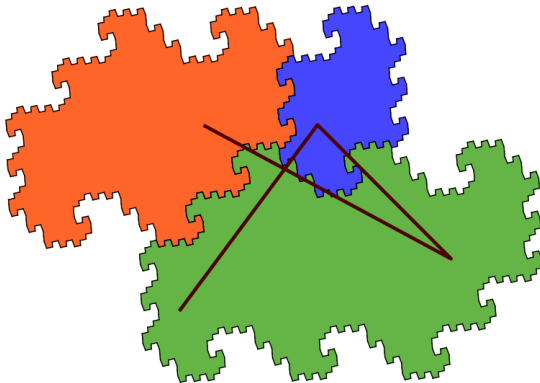
Orbite : 213



# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

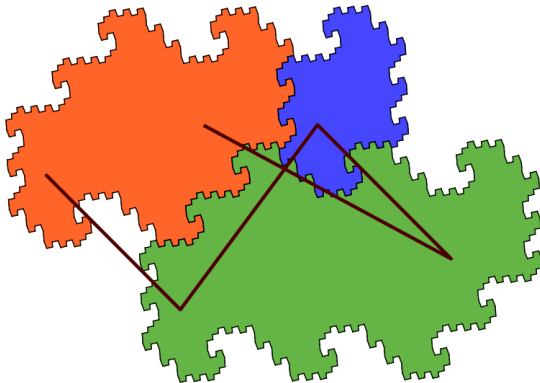
Orbite : 2131



# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

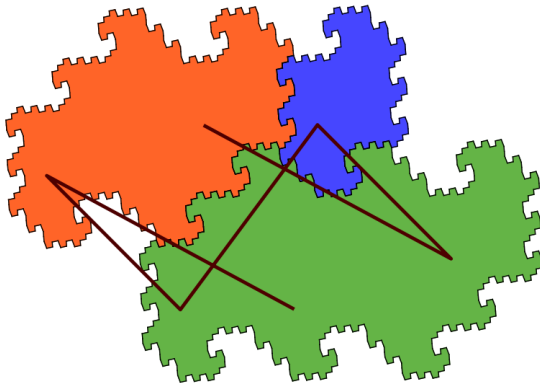
Orbite : 21312



# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

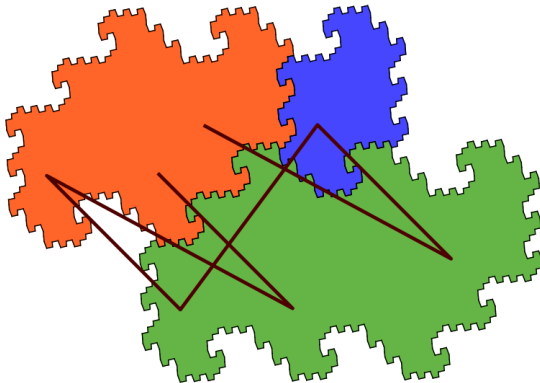
Orbite : 213121



# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

Orbite : 2131212

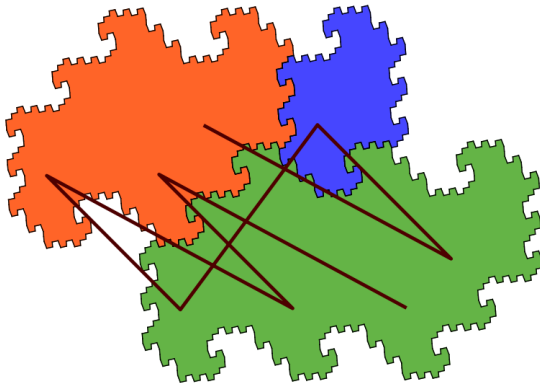




# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

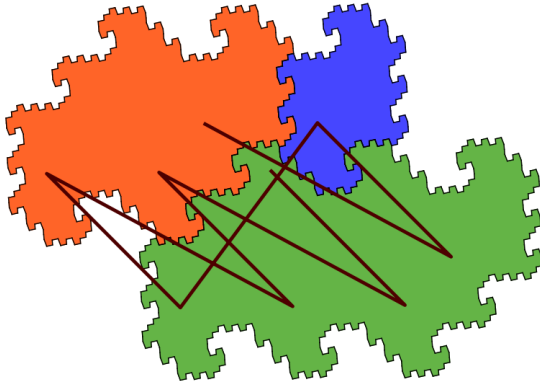
Orbite : 21312121



# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

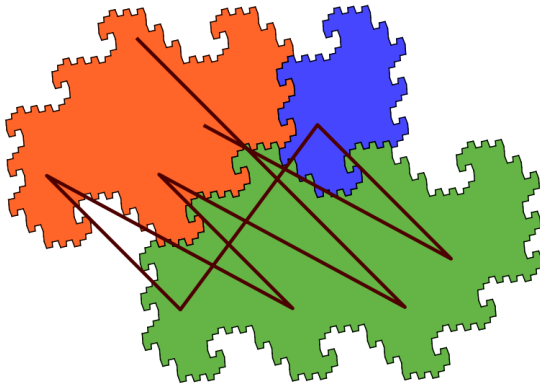
Orbite : 213121211



# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

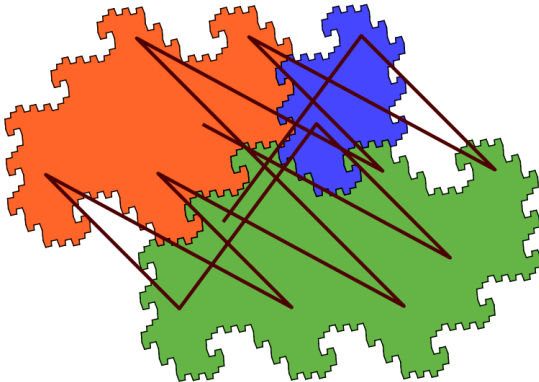
Orbite : 2131212112



# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

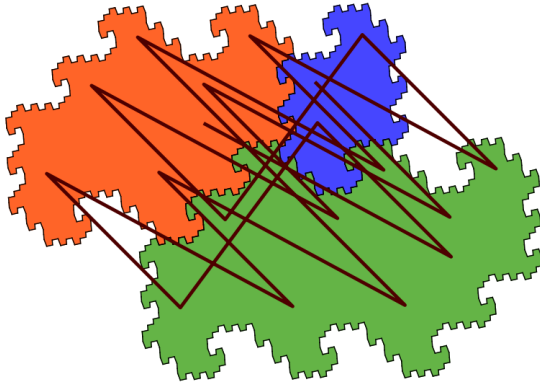
Orbite : 213121211212131



# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

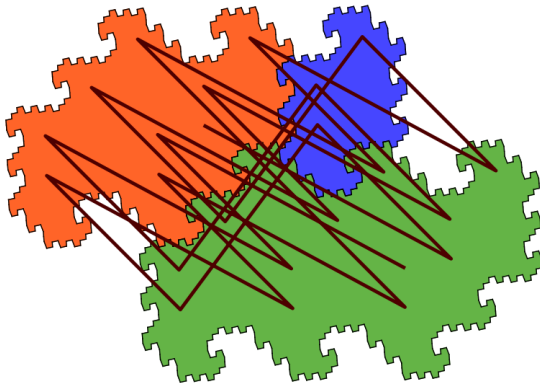
Orbite : 21312121121213121213



# Dynamique de l'échange de morceaux

$1 \mapsto 12$ ,  $2 \mapsto 1312$ ,  $3 \mapsto 112$

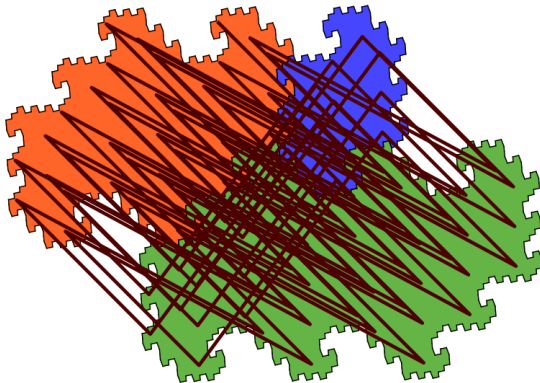
Orbite : 2131212112121312121312121



# Dynamique de l'échange de morceaux

$1 \mapsto 12, 2 \mapsto 1312, 3 \mapsto 112$

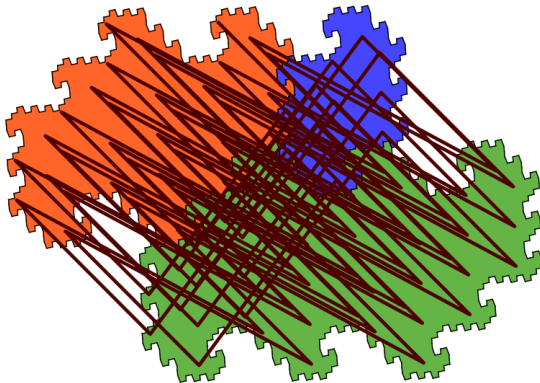
Orbite :  $\cdots 2131212112121312121312121 \cdots \in S_\sigma \subseteq \{1, 2, 3\}^{\mathbb{Z}}$



# Dynamique de l'échange de morceaux

$1 \mapsto 12, 2 \mapsto 1312, 3 \mapsto 112$

Orbite :  $\cdots 2131212112121312121312121 \cdots \in S_\sigma \subseteq \{1, 2, 3\}^{\mathbb{Z}}$



$(S_\sigma, \text{décalage d'une lettre}) \cong (\text{morceaux}, \text{échange de morceaux})$



## **2. Substitutions 2D**

$$1 \mapsto \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$2 \mapsto \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$1 \mapsto \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$2 \mapsto \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

1

$$1 \mapsto \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$2 \mapsto \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$1 \mapsto \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$1 \mapsto \begin{matrix} 1 & 2 \\ 2 & 1 \end{matrix}$$

$$2 \mapsto \begin{matrix} 2 & 1 \\ 1 & 2 \end{matrix}$$

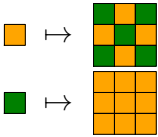
$$1 \mapsto \begin{matrix} 1 & 2 \\ 2 & 1 \end{matrix} \mapsto \begin{matrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 \end{matrix}$$

$$1 \mapsto \begin{matrix} 1 & 2 \\ 2 & 1 \end{matrix}$$

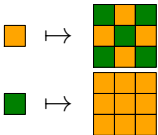
$$2 \mapsto \begin{matrix} 2 & 1 \\ 1 & 2 \end{matrix}$$

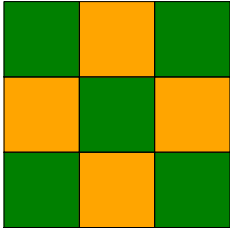
$$1 \mapsto \begin{matrix} 1 & 2 \\ 2 & 1 \end{matrix} \mapsto \begin{matrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 \end{matrix} \mapsto \begin{matrix} 1 & 2 & 2 & 1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & 2 & 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 & 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 & 2 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & 2 & 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 & 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 & 2 & 1 & 1 & 2 \end{matrix}$$

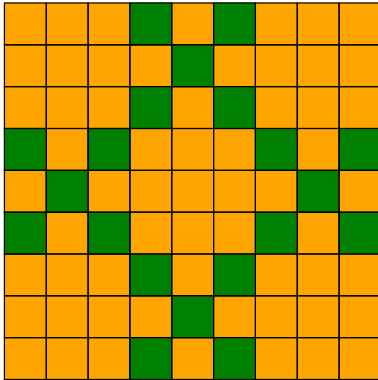


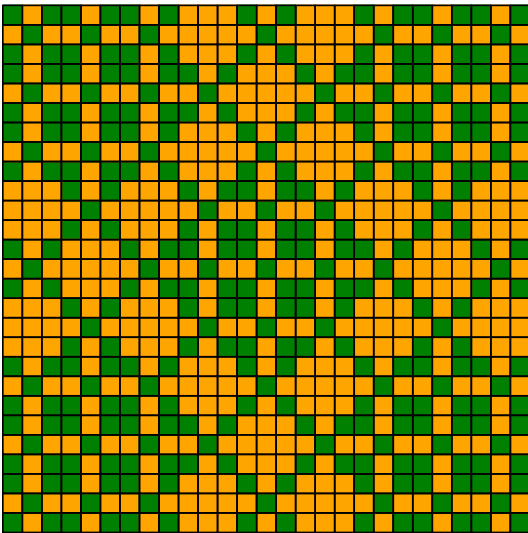


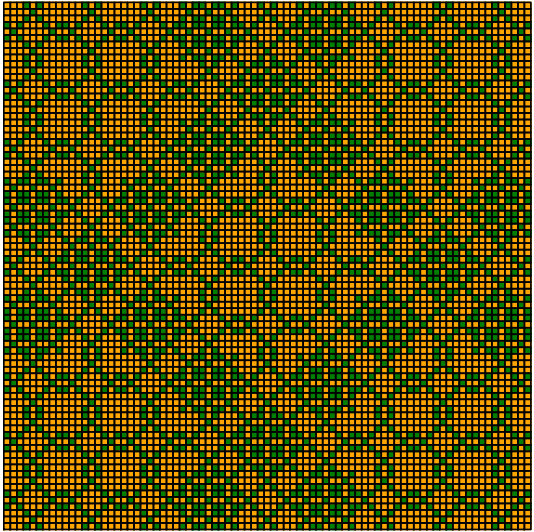
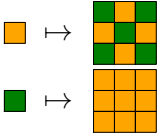












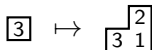
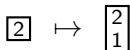
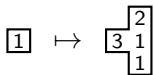
$$\boxed{1} \mapsto \begin{array}{|c|} \hline 2 \\ \hline \boxed{3} \quad 1 \\ \hline 1 \\ \hline \end{array}$$

$$\boxed{2} \mapsto \begin{array}{|c|} \hline 2 \\ \hline 1 \\ \hline \end{array}$$

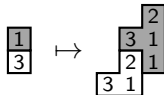
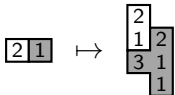
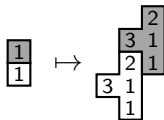
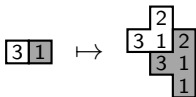
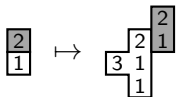
$$\boxed{3} \mapsto \begin{array}{|c|} \hline 2 \\ \hline \boxed{3} \quad 1 \\ \hline \end{array}$$

$$\boxed{1} \mapsto \begin{array}{|c|} \hline 2 \\ \hline 3 & 1 \\ \hline & 1 \\ \hline \end{array} \quad \boxed{2} \mapsto \begin{array}{|c|} \hline 2 \\ \hline 1 \\ \hline \end{array} \quad \boxed{3} \mapsto \begin{array}{|c|} \hline 2 \\ \hline 3 & 1 \\ \hline \end{array}$$

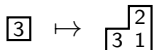
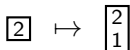
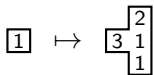
Comment “concaténer” ? !



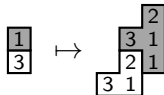
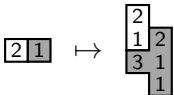
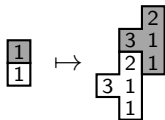
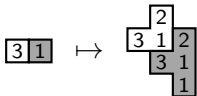
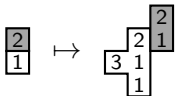
Comment “concaténer” ? !



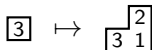
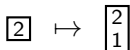
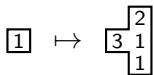




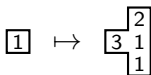
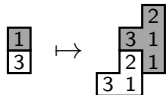
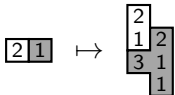
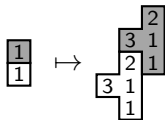
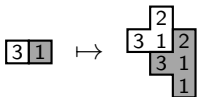
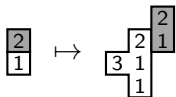
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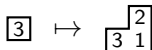
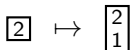
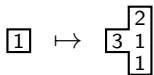


$\boxed{1}$

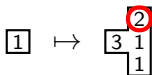
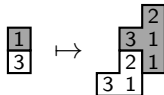
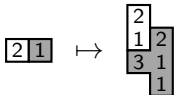
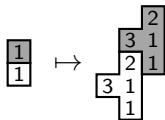
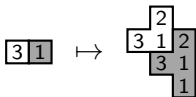
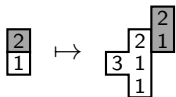


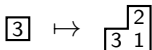
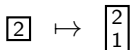
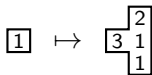
Comment “concaténer” ? !



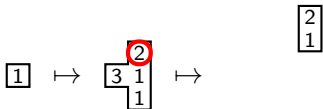
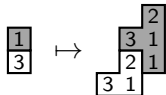
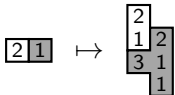
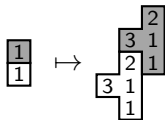
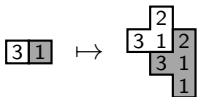
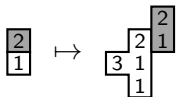


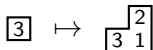
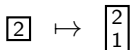
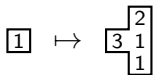
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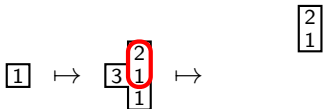
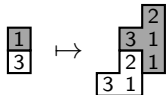
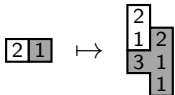
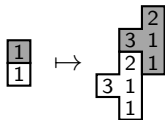
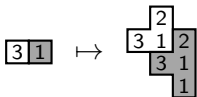
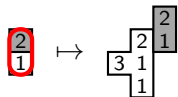


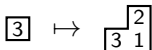
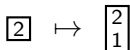
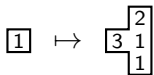
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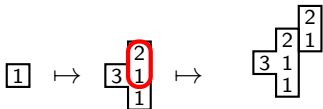
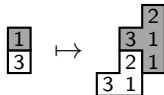
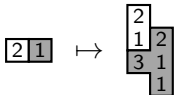
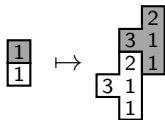
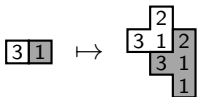
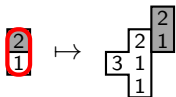


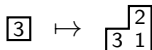
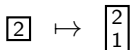
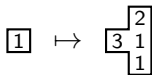
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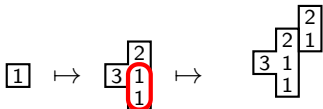
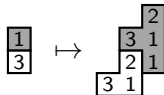
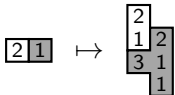
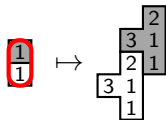
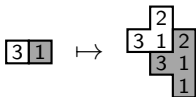
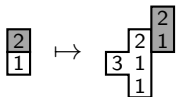


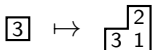
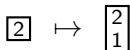
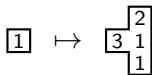
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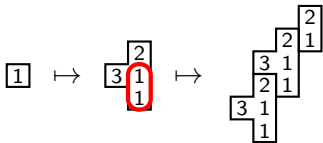
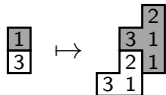
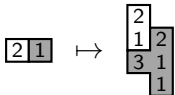
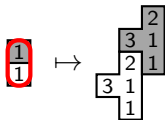
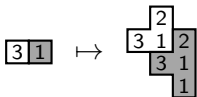
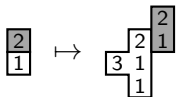


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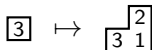
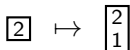
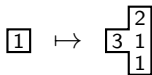




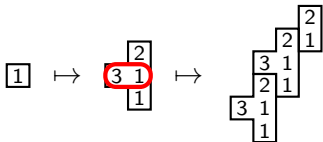
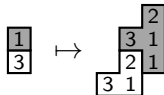
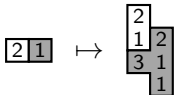
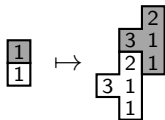
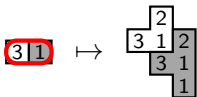
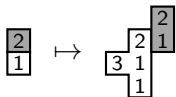
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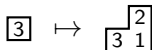
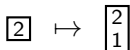
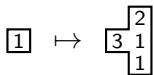




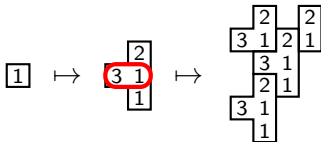
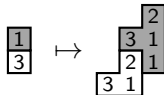
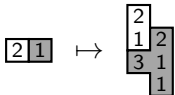
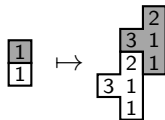
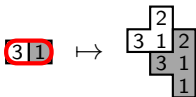
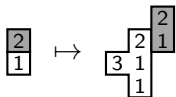


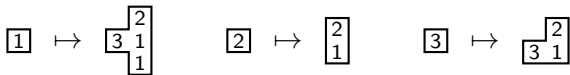
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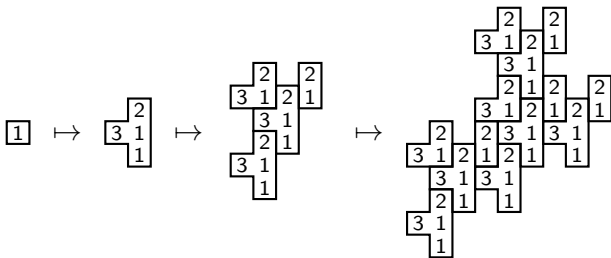
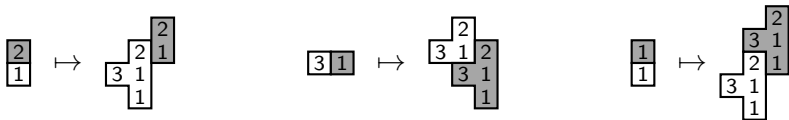


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- ▶ la substitution est **bien définie** (une seule image possible) ?

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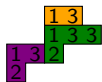
Ces deux propriétés sont en fait **indécidables** !

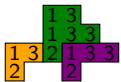
1 3  
2

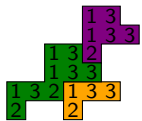
1 3 3  
2

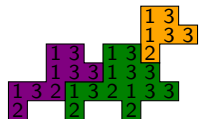


1	3	
1	3	3
2		

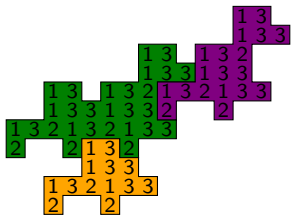






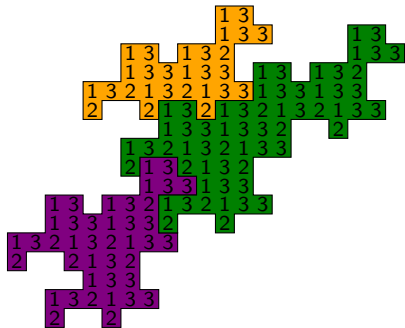


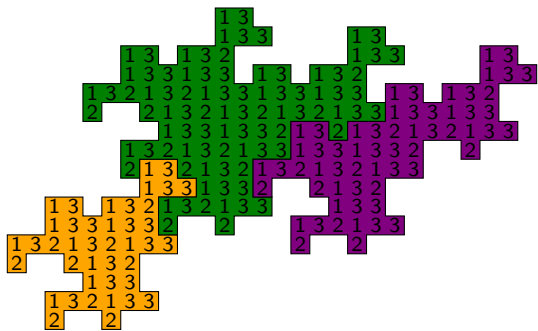


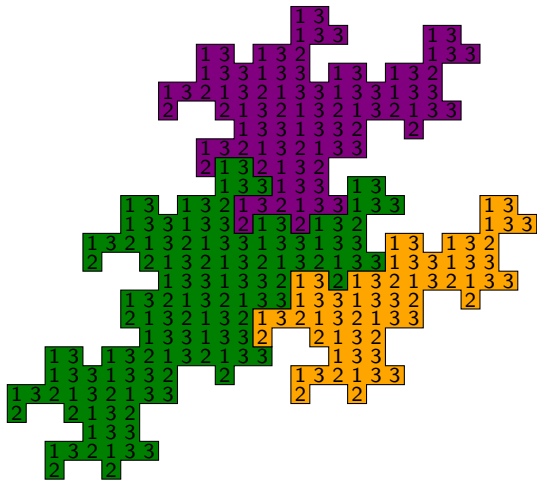


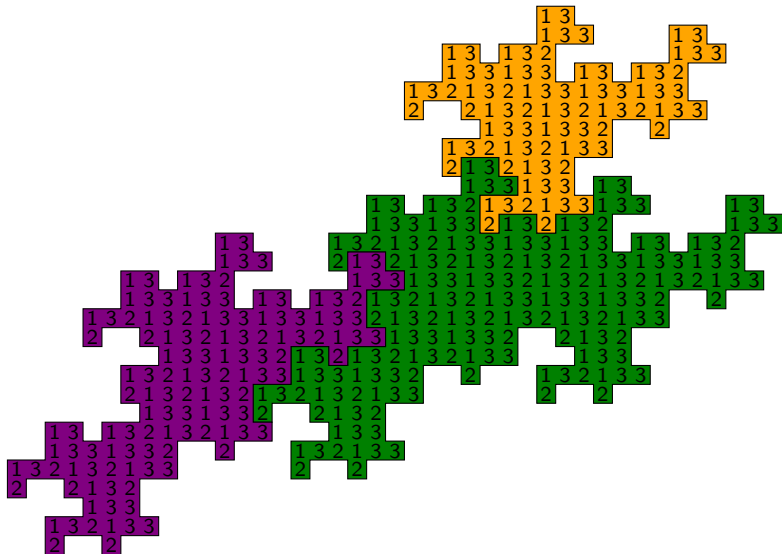


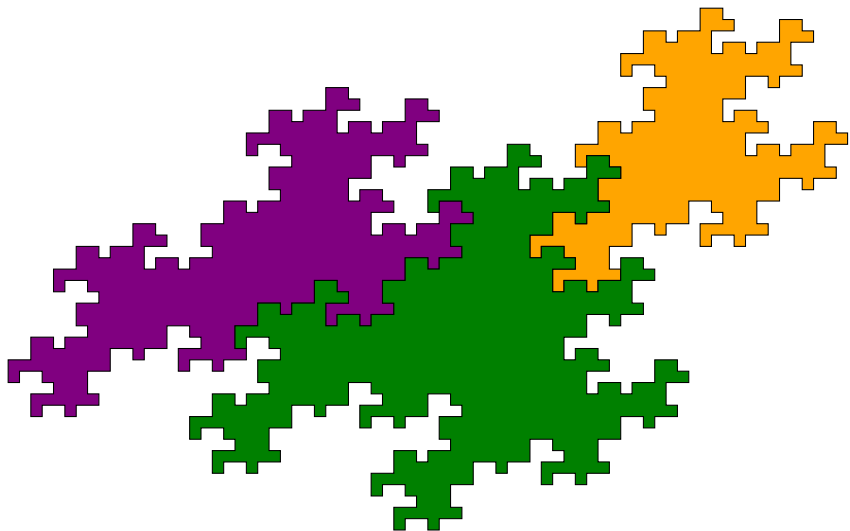












**À table !**

